Spinners, scroll bars and Simpson's rule

Ed Staples
Erindale College, ACT
<ed.staples@erindalec.act.edu.au>

One of the most remarkable devices embedded in a Microsoft Excel spreadsheet is known as the *spinner*. Its staggering simplicity is undoubtedly its strength. As an incrementing device that allows graphs to dance across the screen, it gives the concept of variability a whole new meaning. Spinners and their close cousins *scroll bars* can be grabbed from the view menu (under tools, forms), placed tactfully in amongst your equations, and playfully put to work.

This article is one such investigation using the little critters, but there are many more. A detailed description of the spinner can be found at www.canberramaths.org.au and examples were demonstrated at a workshop given at AAMT's biennial conference in Brisbane, January 2003.

Thomas Simpson (1710–1761) devised a method for finding areas under curves using lengths of ordinates drawn from a base line. Generally the more ordinates, the more accurate are the answers. Simpson's rule is a fairly standard introduction to the fundamental theorem of Calculus, but teachers are equally comfortable with using it as a postscript to integration, justifying its inclusion as an escape clause for functions whose integrals have difficult or non-existent closed forms.

One of the problems with using Simpson's rule as an introduction to integral calculus is that the proof of the remarkably simple formula depends on knowledge of integration of quadratic functions. The cleverness of the procedure lies in approximating the renegade curve with a well-behaved parabola. Of course we knew this; and we also knew that if the curve under investigation is a parabola itself, then Simpson's rule gives the exact answer.

Less well known is that for a polynomial curve of degree 3, Simpson's rule delivers perfect results again. This turns out to be an interesting factoring exercise using the rule with three ordinates and the general third degree polynomial function, and I have outlined a proof at the end of the article. Higher degree polynomials fail to deliver such niceties.

The following graphs are snapshots of an excel program with lots of imbedded spinners. The program has been placed on www.canberramaths.org.au. The scatterings of spinners at the top are control-

ling variability on the coefficients of a polynomial of degree five or less. Clicking on these winds the coefficients up and down, and causes the hopelessly dependent sketch to obey. Of course two other spinners are restricting the domain.

Overlaid on the curve of interest is the Simpson's parabola formed on three of the curve points, two at each end and one in the middle.

Here is our first graph. It is the cubic $y = 0.5x^3 - 2.5x^2 + 5x + 20$. The parabolic arc together with its equation is shown (using trend lines on scatter plots) in Figure 1.

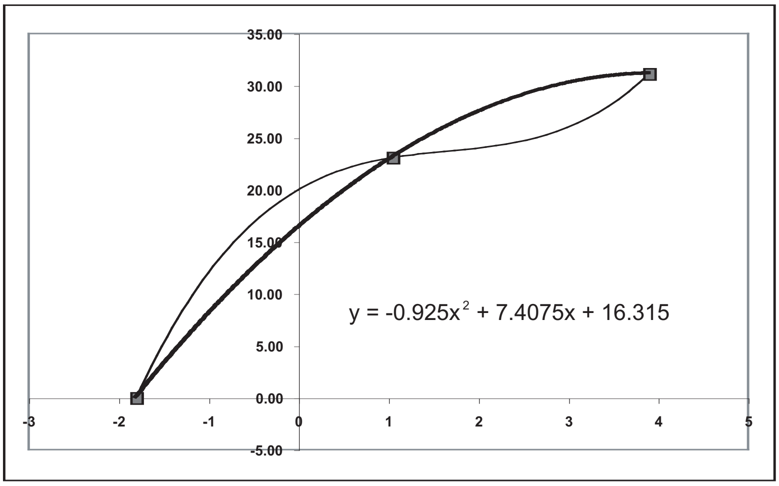

Figure 1

Between $x = -1.8$ and 3.9 the area under both curves and above the x-axis is 117.2 square units. You can see that the two enclosed sections must be equal. The graph is illuminating.

As we throw the coefficients around, Simpson's parabola responds brilliantly, changing its acuteness and concavity to keep in perfect step with the cubic. The dance is simply stunning!

Here is another position (Figure 2) with a net area of 4.8 square units. The cubic is $y = 3x^3 - 15x^2 + 5x + 20$.

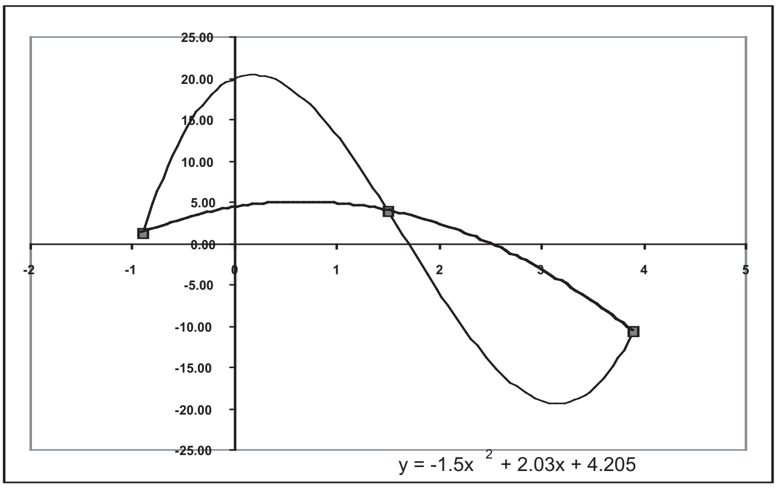

Figure 2

One little serendipity with this program was the discovery that the roots of a cubic could be approximated by the overlay of Simpson's parabola. To illustrate the concept, suppose we were interested in finding out the middle root of the cubic shown above. It lies somewhere between $x = 1$ and $x = 2$.

Why not restrict the domain of the cubic to those boundaries? The resulting graph is shown below, in Figure 3.

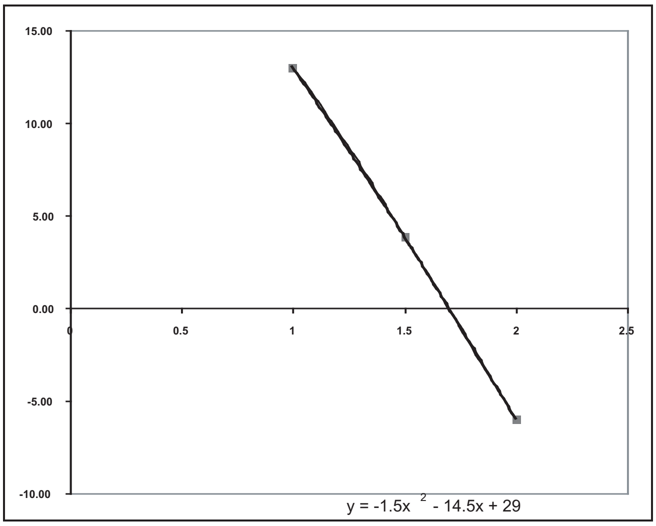

Figure 3

Now Simpson's parabola appears to lie directly over the cubic, so why not use the parabola to find the root? Using the equation shown, the root of the cubic lies close to the root of the parabola. That is, lies close to

$$\frac{\sqrt{1537}-29}{6}$$

or about 1.7008.

If we move into higher degree polynomials by incrementing the coefficients of x^5 and x^4, we dynamically see Simpson's parabola fail to give the exact answer.

Now for the promised proof.

For the polynomial function $f(x) = ax^3$, the area bounded by $x = h$ and $x = k$, the x-axis and the curve, according to Simpson's rule, is given by:

$$A = \frac{k-h}{6}\left(f(h) + 4f\left(\frac{k+h}{2}\right) + f(k)\right)$$

Substituting the function values, we have

$$A = \frac{k-h}{6}\left(a\left(h^3+k^3\right) + 4\left(\frac{a\left(k+h\right)^3}{2}\right)\right)$$

$$A = \frac{k-h}{6}\left(\frac{2ah^3+2ak^3+a\left(k+h\right)^3}{2}\right)$$